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Inelastic Seismic Response of R/C Structural Systems in terms of Hysteresis Modelling in Overall Sense: State of the Art

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Abstract

Seismic performance of reinforced concrete(R/C) membered structure during earthquakes has been differed significantly from their intended behavior due to progressive loss of strength and stiffness. In this context, from the macro scale point of view, the present study aims to provide an overview of hysteresis model development to assess progressive seismic damage of an entire structural system. Considering the requirement of number of input parameters and complexity to solve the corresponding equations, this study presents a state of the art review of different modelling philosophies by grouping them in i) simpler hysteresis model and ii) complex hysteresis model. It is found out that in comparison with complex hysteresis models, which require case-specific detailed calibration study, simpler hysteresis model on the other hand can predict the behavior of R/C structural elements with progressively degraded strength and stiffness in approximate yet realistic manner, which is more acceptable to predict the overall behavior of structures. Recognizing the same, simpler hysteresis models are emphasized in this study. Significant number of hysteresis models have been analyzed to highlight previous drawbacks, subsequently a simpler enhanced hysteresis model has also been proposed with greater accuracy taking care of non-specificness, computational efficiency and mathematical tractability.

Keywords

Inelastic seismic behavior, Progressive seismic damage, Simpler hysteresis modelling, Strength and stiffness degradation

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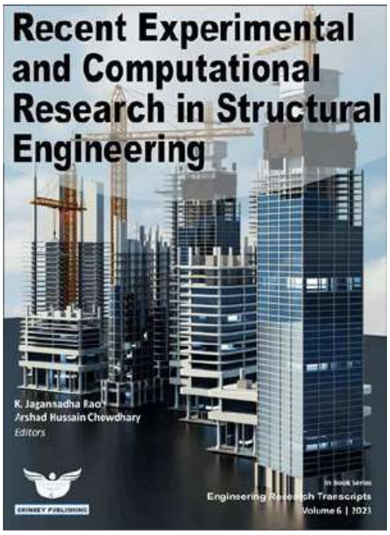


1. Introduction

Since past few decades increase in the losses due to natural catastrophes has been reported worldwide. Among other catastrophic incidents, earthquake is the most disastrous one. Past experience of human causalities and huge economic loss due to destruction of reinforced concrete (R/C) structures during strong ground shaking, make whole world to concern about the vulnerability of RC structures. Among remarkable catastrophic incident some are reported as – (Bhuj Earthquake 2001 -Gujrat India; Nepal Earthquake 2015-Nepal; Iran- Iran Earthquake 2017 – Iran; Albania Earthquake 2019 – Albania, Durres; Haiti Earthquake 2021 – Haiti, Nippes; Afghanistan Earthquake 2022 – Afghanistan, Khost, etc). Hence, now a day seismic behaviour of structure due to repeated reversible loading became a topic of active research. Seismic performance of R/C membered structure during earthquakes has been differed significantly from their intended behaviour due to progressive loss of strength and stiffness in structural members suffered by post elastic range loading. Structures subjected to earthquake excitation undergoes repeated cyclic deformation creates invariably deterioration in the hysteretic characteristics which must be taken into account for seismic-designing. However, a realistic estimate of seismic damage can be made with a suitable hysteresis modelling with progressive degradation of strength and deterioration of stiffness characteristics at all level. In this backdrop, this study is to gather and apprehend the existing hysteresis models which can capture progressive seismic damage to give an overall behaviour of the structure during seismic excitation. Out of several previously developed hysteresis model, some significant hysteresis models are available in the literature and are not limited to PEER Structural Performance Database, [1]; [2]; [3]; [4]; [5]; [6]; [7]; [8]; [9]; [10], etc. Based on the literature survey conducted in this chapter, hysteresis modelling has been categorized in 1) Simplified Hysteresis modelling and 2) Complex hysteresis modelling. The division of Simplified Hysteresis model and complex hysteresis model is done by recognizing the input parameters needed and complexity to solve the corresponding equation.

1.1 Research significance

This objective of the paper is to comprehend and evaluate existing hysteresis models on the basis of performance in predicting progressive seismic damage. The present paper is an effort to illustrate all significant aspects of the performance of hysteresis models to predict the overall behavior of a R/C membered structure and not to present behavior of a particular R/C structural member which are primarily able to predict pre-determined laboratory based cyclic loading experimental results in a comparatively accurate manner. The substantive findings of this study can assist researchers and engineers in predicting R/C structural responses considering progressive seismic damage in a simplified way.

1.2 Review of existing hysteresis models

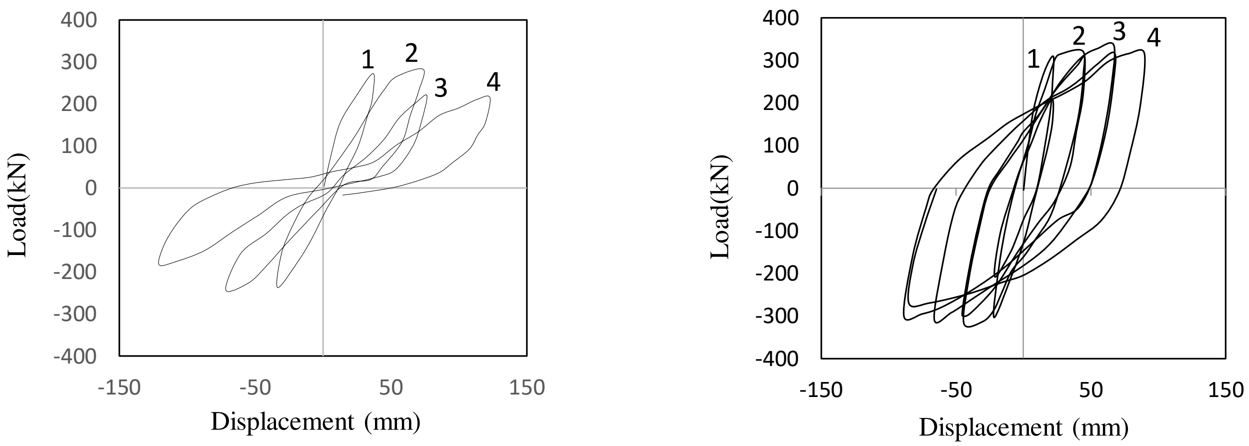
To assess seismic damage as mentioned earlier by grouping existing hysteresis models in i) Simpler hysteresis models and ii) Complex hysteresis models, a brief review on the selective significant hysteresis models are presented in this section.

For the purpose of the study of general hysteresis behaviour, two experimental load-displacement curves under cyclic reversal loading has been considered are shown in Figure-1 first one is opted from Lehman et al.(2002), Specimen- CD15-1450. [11] and second one by Saatcioglu, M; Ozcebe, G (1989) [5].

The following are the most noticeable hysteretic characteristics:

(i) Due to the flexural cracking of concrete and the longitudinal reinforcement yielding, both cases show continuous changes in stiffness.

(ii) The loading stiffness was noticeably lower in the second cycle after a load reversal was repeated to reach the newly attained maximum deformation amplitude. This progressive decrease of stiffness continues cycle after cycle. Resistance after yielding in the case of load displacement curve 1(a) is significantly



a) Experimental load-displacement curve reported in [11]

b) Experimental load-displacement curve reported in [5]

Fig. 1. Sample experimental load-displacement curves.

changed in each cycle. On the other hand, the resistance after yielding in load-displacement curve of Figure1(b) is almost identical (cycles 2, 3 and 4).

(iii) In both cases and in both directions, yield strength in each cycle is different. The Figure-1 clearly demonstrates a drop of yield strength from about 220kN in the first cycle of loading to about 170kN in the last cycle of loading and a drop of yield strength from about 330kN to about 270kN in load-displacement curve Figure1(a) and Figure1(b) respectively. Like stiffness, progressive decrease of yield strength is noticeable as cycle increases.

(iv)The sudden decrease in stiffness also known as pinching which is result of crack closure is observed in both cases. A low pinching in the reverse loading branch is observed in case of Figure-1(b) whereas heavy pinching is observed in Figure-1(a).

(v) A reinforced concrete's hysteresis characteristics depends on its loading history, and

(vi) For the two successive cycles of flexural behavior of the member, the peak deflection resistance is nearly the same.

1.2.1 Simpler hysteresis models

Several investigations on hysteresis models have been carried out in the past since 1960. Among them elastic-plastic model has been proposed in [12] Figure-2, where the primary force deformation curve is represented by an elastic portion. Changes of stiffness after yielding, drop of yield strength and pinching is not considered by the model. The variables considered are F_y: Yield strength, U_y: Yield displacement, U_m: Maximum displacement and K_e: Elastic stiffness in which input parameters are K_e and F_y.

As shown in Figure-3, Clough and Johnston(1966) [14] proposed a bilinear primary curve with ascending post-yielding branches to represent the hysteresis behavior of a reinforced concrete beam-column sub-assemblage. The post yielding stiffness K_p is defined as $K_p = \alpha K_e$: where K_e = Initial elastic stiffness and α = stiffness ratio, the unloading stiffness $K_u = K_e$ but the reloading was aimed towards the maximum displacement of the previous cycle. Mahin and Betero (1976) [15] modified the Clough model by incorporating a reduction in unloading stiffness K u along with a maximum displacement as follows:

$$K_u = K_p \left(\frac{U_m}{U_y} \right)^{-\alpha} \tag{1}$$

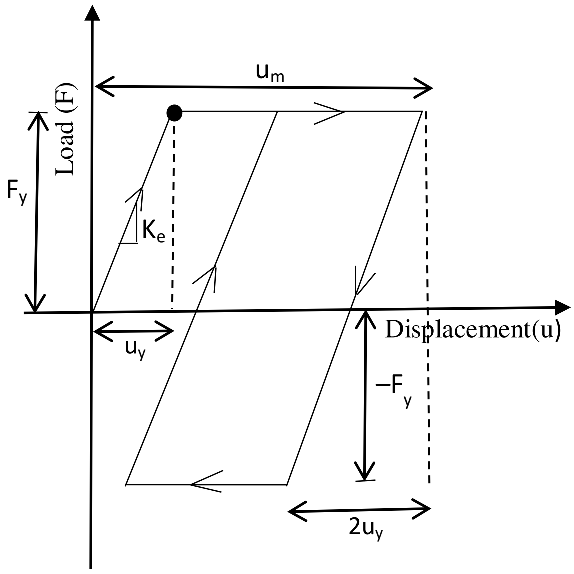


Fig. 2. Bilinear model reported in [13]

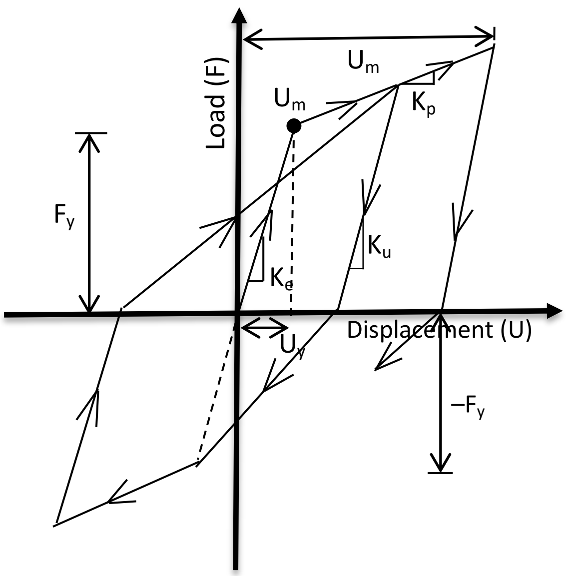


Fig. 3. Hysteresis model by Clough and Johnston (1966)[16]

The variables considered in Clough model are: K_e, K_p, K_u, U_m, U_y. In which K_u = Unloading stiffness, and other implies the same as of previous reviewed model. Input parameters involved in this model are: K_e and U_y.

There is an unrealistic feature of the Clough model that is revealed by Mahin and Bertero (1976) [15] and Riddell and Newmark (1979) [17] that after small unloading, the model reloads unrealistically toward maximum deformation when experiencing large load reversals followed by small load reversals. The model was modified to reload along the same unloading branch until it reached the reloading branch, then aim for peak deformation. Mahin and Bertero (1976) added additional flexibility to the model by including a positive post-yield stiffness and variable unloading stiffness as a function of peak deformation. This model is referred to as the modified-Clough and has been widely used to simulate the behavior of flexural controlled reinforced concrete elements.

More complex tri-linear primary curve with degrading stiffness representing un-cracked, cracked and post-yielding stages and initiation of non-linear deformation after section cracks has been proposed by

Takeda et al.(1970) [18] shown in Figure-4. As a result of its compatibility with computational programming and extensive application in earthquake engineering to research the seismic response of R/C structures, Takeda's model has become a very dominating model for inelastic structural analysis of R/C systems. [19].

The unloading stiffness K_u is calculated in terms of initial elastic stiffness, yield displacement (Uy) and maximum displacement (U_m) in the form:

$$K_u = K_y \left(\frac{U_y}{U_m} \right)^{0.4} \tag{1}$$

Like Clough and Johnston model reloading branch projects towards the previous unloading point to produce decrease of stiffness. Other than previous reviewed model, the only variable involved in Takeda's model is: K_y = stiffness of the load path which is joining the yield point in one direction and crack point on the other direction.

Otani and Sozen (1972)[20] suggested changing the Takeda model, replacing the tri-linear initial loading portions with a bilinear connection. Bi-linear Takeda model is the name of the ensuing model [21].

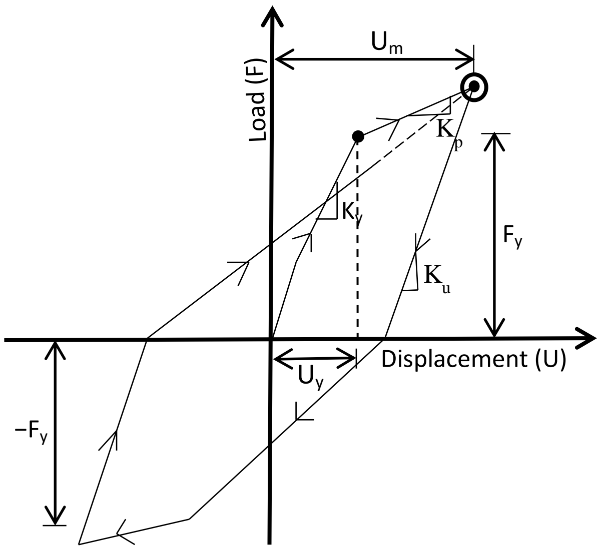


Fig. 4. Hysteresis model by Takeda et al. (1970)[21]

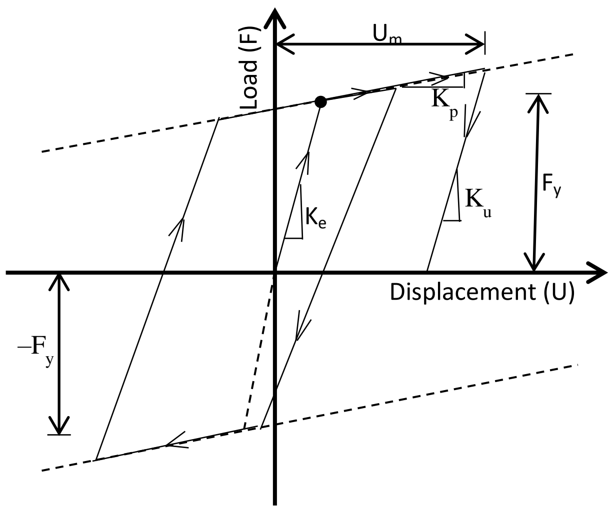


Fig. 5. Hysteresis rule by Imbeault and Neilson (1973)[22]

The stiffness changes only when the prior maximum is surpassed in any direction, as illustrated in Figure 5, according to a stiffness degrading bilinear model proposed by Imbeault and Neilson (1973) [22]. Saidi &

Sozen (1979)[23] developed a model (Q-hysteresis model) nearly similar to Clough and Johnstone[16] as shown in Figure-6.

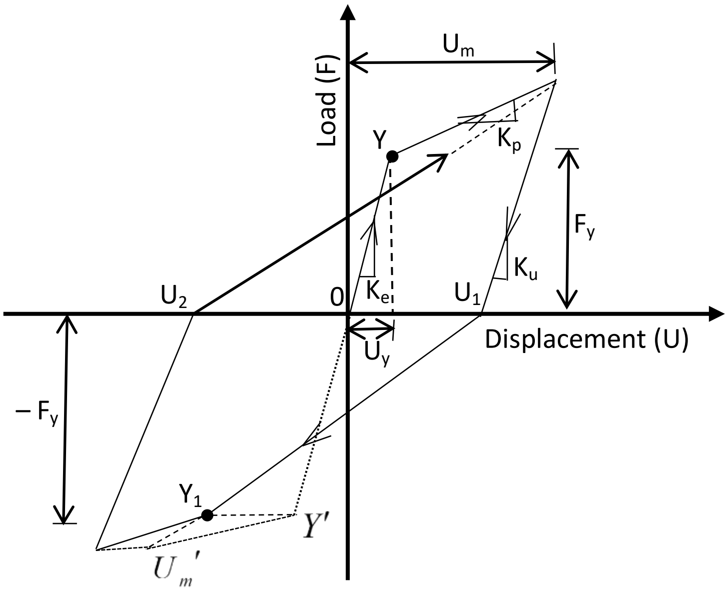


Fig. 6. Q- hysteresis model by Saiidi and Sozen (1979) [24]

Reloading stiffness is determined as slope of the line $U_1 - U_m'$ with U_m' being the point on the primary curve symmetric to U_m with respect to origin. The unloading stiffness (K_u) is calculated by the following expression –

$$K_u = K_e \left(\frac{U_y}{U_m} \right)^{\alpha} \tag{2}$$

Where the "α" is the stress ratio of constant value and other notations implies same as of previous reviewed models.

Two straightforward hysteresis models are put forth in a research by Das and Dutta (2002) [25] that can easily account for stiffness and strength deterioration characteristics. Among those two simple hysteresis models, the relative accurate model comprises only three input parameters namely initial elastic stiffness K_e, initial yield strength F_y and rate of strength degradation δ. Stiffness deterioration of reloading branch is calculated by following the principle of Takeda's model i.e.- targeting the prior location of unloading of the same side. Details of the model are shown in Figure- 7.

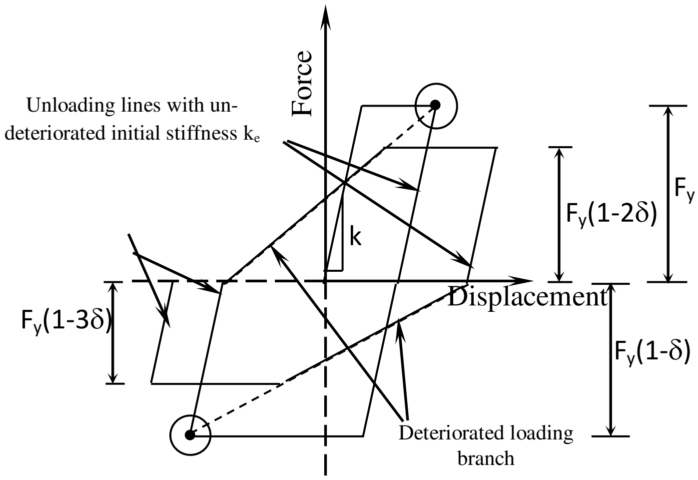


Fig. 7. Hysteresis model by Das and Dutta (2002) [25]

Ibarra et al. [26] presented a pinching hysteresis model with three control points namely, yield strength F_y, peak strength F_{max}, residual strength ($F_{residual} = \lambda F_y$), and the respective displacements are D_y, D_c and D_r as shown in Figure-8, K_e, K_s and K_c are elastic stiffness, post yield stiffness ($K_s=\alpha_s K_e$) and post-capping (negative) stiffness ($K_c=\alpha K_e$), respectively and α_s, α_c and λ are constants. Figure-9 shows the pinching hysteresis model without deterioration.

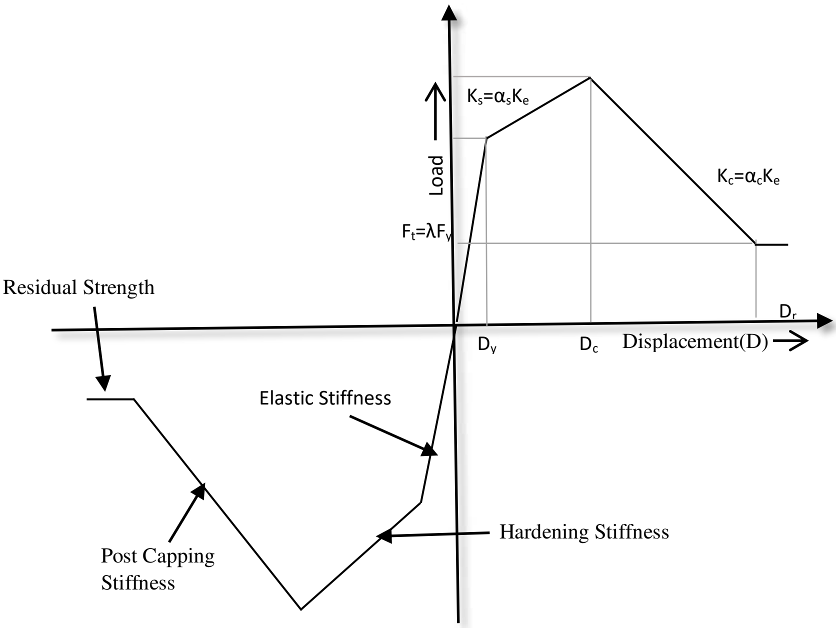


Fig. 8. Backbone curve of the Ibarra et al. pinching hysteresis model. [26]

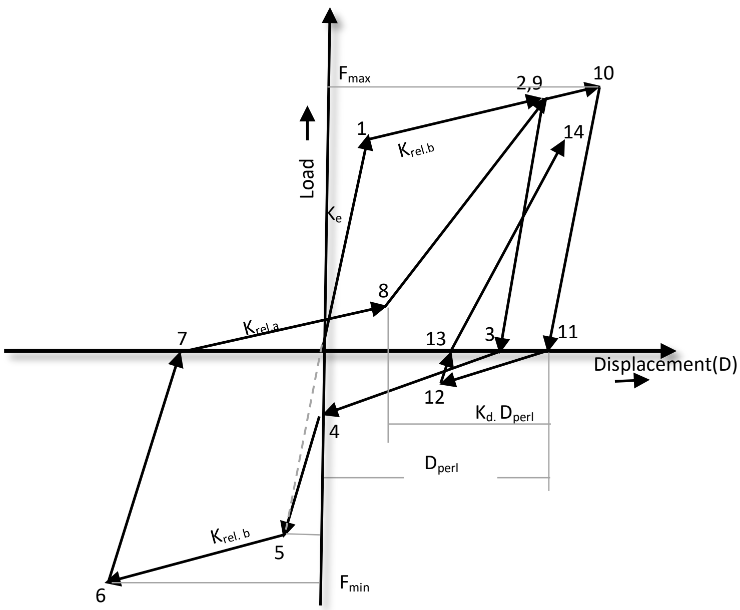


Fig. 9. Ibarra et. Al. [26] load deflection diagram for the pinching hysteresis model without deterioration.

In a recent study by Hazra and Das (2023)[27] as shown in Figure-10 has enhanced the superior model of Das and Dutta(2002)[25], by incorporating strain-hardening effect and the effect of pinching. To encounter the drawbacks of not considering strain hardening and pinching an additional parameter stiffness ratio ($\boldsymbol{\alpha}$) is introduced. Strain hardening after yielding is introduced by multiplying the loading stiffness by fixed parameter $\boldsymbol{\alpha}$ up to unloading point. Additional stiffness deterioration due to pinching in loading branch is introduced also by multiplying the stiffness ratio ($\boldsymbol{\alpha}$) up to displacement 0 (zero) level to the loading stiffness. The assumption made about the sudden stiffness change of stiffness or pinching in reverse-loading branch is by recognizing actual crack closing behavior of a R/C element in which cracks start to close in reverse-loading branch and completely close when displacement is zero (0).

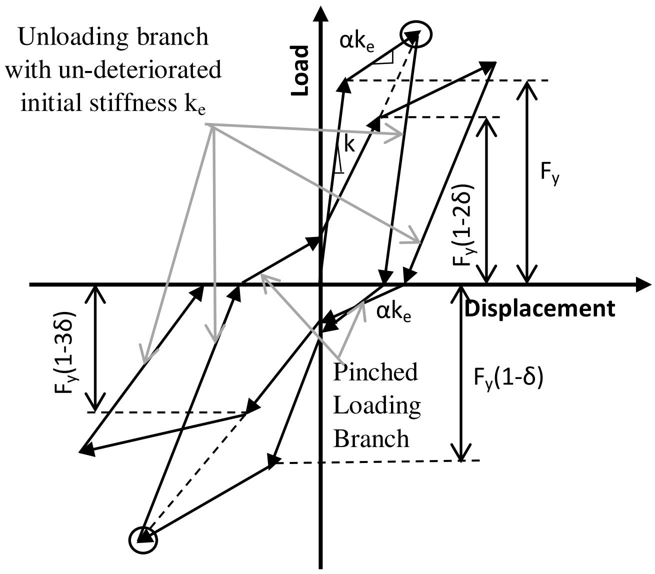


Fig. 10. Developed Model with incorporation of Pinching effect and Strain Hardening

The reported enhanced simpler hysteresis model gives an easy calculated result which apparently meets the purpose of the present study. However, the parameters seem to very crucial for the hysteresis performance.

1.2.1.1 Sensitivity Analysis

To come to a conclusion, conduction of sensitivity analysis of all parameters related to the presented hysteresis model by Hazra and Das (2023), has been performed. Observation shows that the value of stiffness ratio (α) is found that this parameter is least sensitive from sensitivity analysis as shown in Figure-11, Figure-12 and Figure-13. The sensitivity analysis has been conducted for low pinched, moderately pinched and heavily pinched hysteresis curve. From the outcome of sensitivity analysis, it is obvious for mere change in stiffness ratio, it has negligible effect on overall response of structural system.

1.2.2 Complex hysteresis models

Another hysteresis model that is now in use is the Bouc-Wen model (Bouc 1967; Wen 1976), which was developed by Wen [28] and introduced by Bouc to characterise non-linear hysteresis systems. For a system with a single degree of freedom (SDOF), Boc proposed a complex smooth changing model. Later, by [29] and [30], the model was expanded to include, respectively, stiffness degradation, strength degradation, and pinching effect.

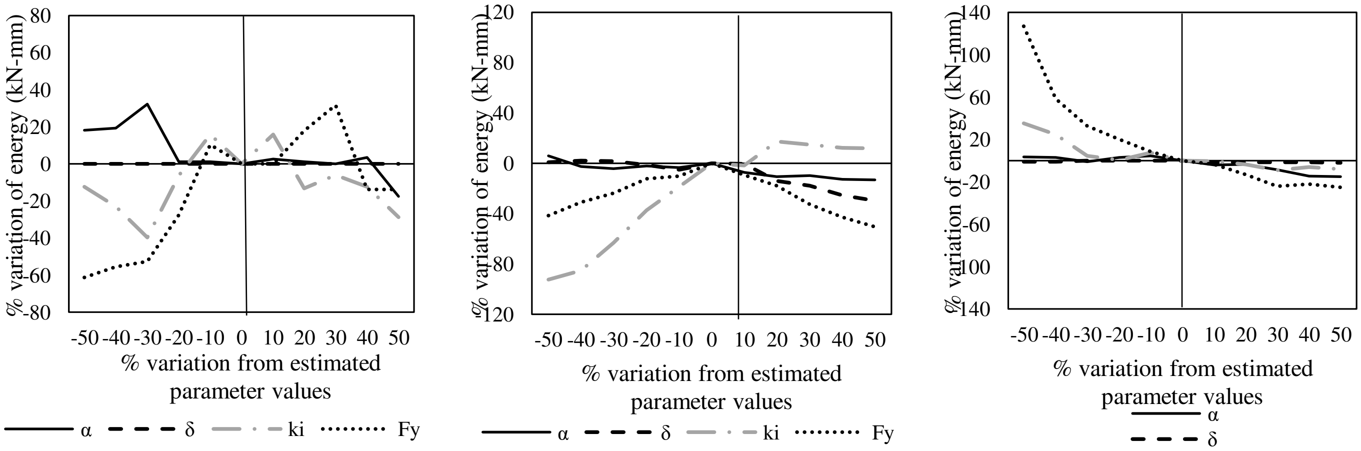
Equation of motion of a Single degree of freedom system is,

$$mu\left(t\right)+cu\left(t\right)+F\left(t\right)=f\left(t\right) \quad (3)$$

Where 'm' denotes mass, u(t) is the displacement, 'c' is the linear viscous damping coefficient, F(t) = restoring force and f(t)= excitation force. Over dot denotes the derivative w.r.t time(t). According to Bouc-Wen model, the restoring force is expressed as,

$$F\left(t\right)=\alpha k_e u(t)+\left(1-\alpha\right)k_e z\left(t\right)$$

(4)Where $\alpha=\dfrac{k_p}{k_e}$ is the ratio of post-yield stiffness (k_p) to pre-yield stiffness (k_e). z(t) is a non-observable hysteretic parameter that follows a nonlinear differential equation with zero initial condition, i.e. – z (0) =0



a) [1] ; Specimen – 88-35-RV 10-60 b) [32]; Specimen – C2-3 c) [5]; Specimen – U6

Fig. 11. Sensitivity of parameters involved in producing analytical curves for lowly pinched Experimetal Hysteresis Curve

$$z(t) = Au(t) - \beta|u(t)||z(t)|^{n-1} z(t) - \gamma u(t)|z(t)|^n \tag{5}$$

$$\text{Or, } z(t) = u(t)\left\{A - \left[\beta \, sign\left(z(t)u(t)\right) + \gamma\right]|z(t)|^n\right\} \tag{6}$$

Where sign denotes signum function A, $\beta > 0$, γ and 'n' are dimensionless quantities controlling the behaviour of the model (n = ∞ retrieves the elastoplastic hysteresis). The restoring force F(t) can be decomposed into an elastic and a hysteretic part as,

$$F^{elastic}(t) = \alpha k_e u(t) \text{ and } F^{hysteresis}(t) = (1-\alpha) k_e z(t) \tag{7}$$

The transition from the elastic to the post-elastic branch is smooth for small values of the positive exponential parameter "n," but abrupt for large values. The hysteretic loop's size and shape are determined by the parameters A, β and γ. Extension of Bouc's model by Wen and later on Bouc (1971) led to a smooth hysteresis model (SHM) that admits stiffness, strength or combined degradation as a function of hysteresis energy dissipation.

Research by [30] discussed a discrete element model for hysteretic behaviour based on the concept proposed in [31]. The restoring force behaviour specified by employing an extended version of the Massing's hypothesis to the initial loading curve $\phi(u)$ over Bou-Wen hysteresis model, which is specified as follows:

$$z = \phi(u) = z_y \left(1 - e^{\frac{-u}{u_y}}\right) \text{ for } u \geq 0 \tag{8}$$

$$\text{And } \phi(-u) = -\phi(u) \text{ for } u < 0$$

$$\text{Where, } z_y = \frac{A}{\beta + \gamma}, \quad u_y = \frac{1}{\beta + \gamma}$$

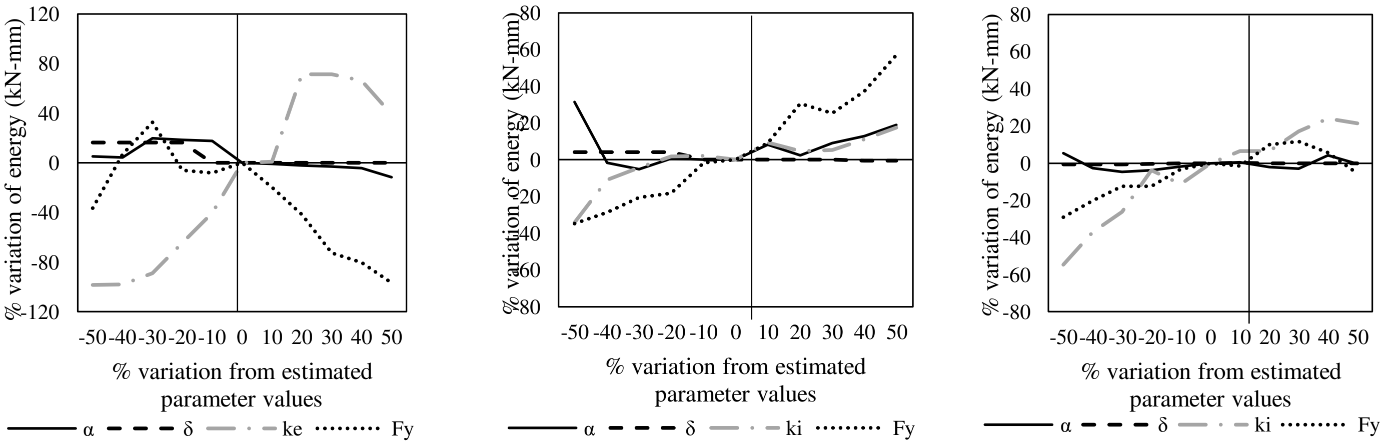
Uy denotes yield displacement of the system. Model parameters are A, β, γ and U_y, which are similar to Bouc-Wen hysteresis model.

Another study by [19] use pivot point in defining degraded unloading stiffness. Hysteresis property achieved through the combination of the tri-linear envelope and only three parameters, which are α, β and γ representing the values of stiffness degradation, strength deterioration and pinching respectively.

2. Prediction of Experimental Load Displacement Curves

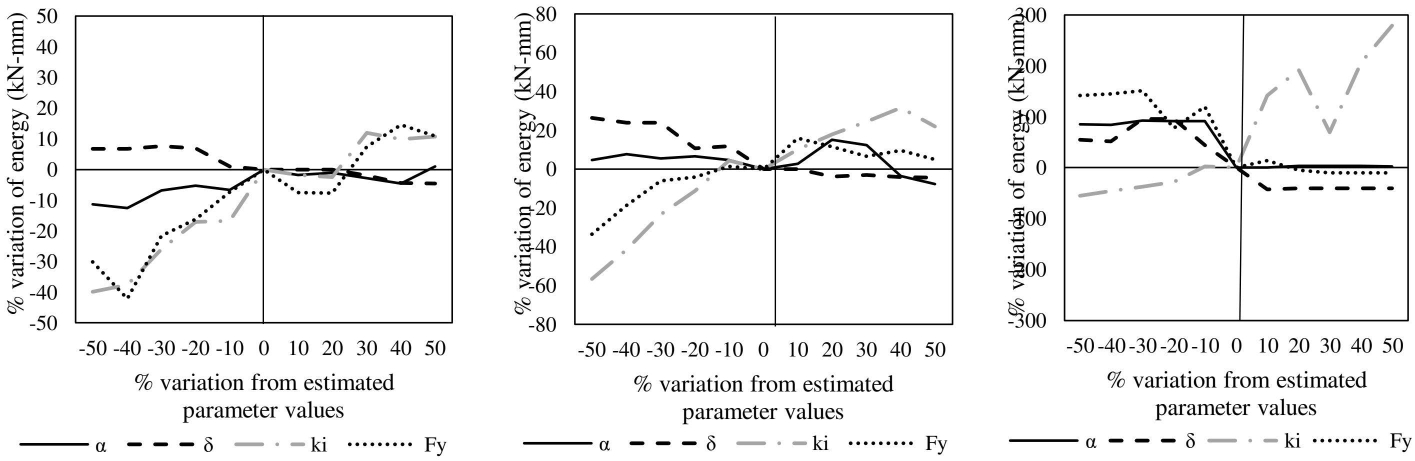
Numerous experimental load-displacement curves for reinforced concrete (R/C) members subjected to cyclic reversal loading are readily available in various sources, including but not limited to [1]; [2]; [3]; [4]; [5]; [6]; [7]; [8]; [9] and [10]. To determine the parameters δ, the number of yield excursions in the experimental load-displacement history is divided by the total strength drop. On the other hand, the parameters Fy, k, and α are directly obtained from the literature curves. For each individual curve, the proposed models are applied separately, utilizing the displacement history, Fy, k, δ, and α as input parameters for computation.

The comparison between the reproduced analytical load displacement curve of the developed model and equivalent experimental curves was conducted to assess their similarity. Figure-2.13 presents the comparison between the proposed model and the previous model [25]. The figure showcases seven different experimental curves along with their corresponding computed curves. The load displacement curves generated computationally using the developed and previous models are depicted as solid red lines in each individual figure. For ease of comparison, the experimental load-displacement curves are overlaid onto each of the computed curves. Upon examining the results, it becomes evident that the curves generated by the



a) [32]; Specimen – C1-3 b) [33] ; Specimen – C1 c) [34]

Fig. 12. Sensitivity of parameters involved in producing analytical curves for Moderately pinched Experimetal Hysteresis Curve

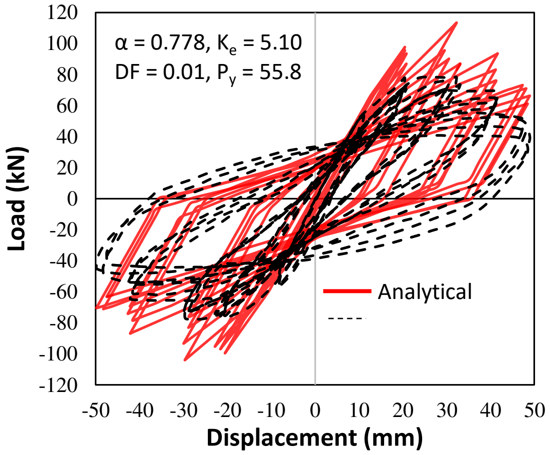


a) [35] b) [11] c) [36]; Specimen – HPRC 10-63

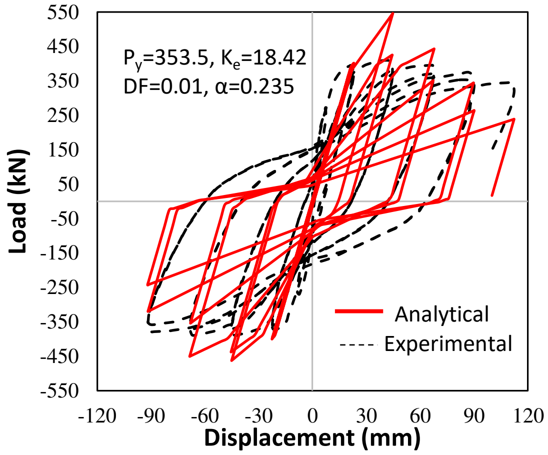
Fig. 13. Sensitivity of parameters involved in producing analytical curves for Heavily Pinched Experimetal Hysteresis Curve

developed model closely resemble the experimental curve. A comparison between the previous model by

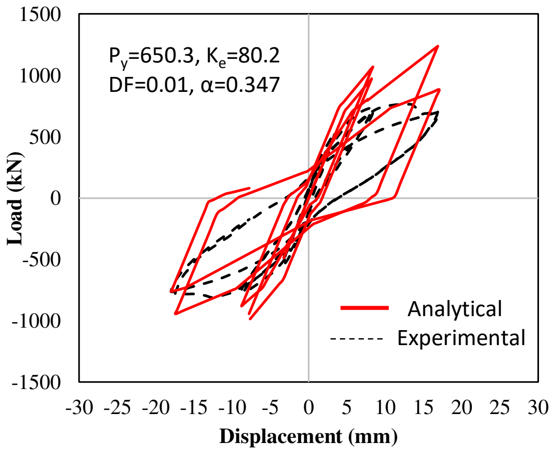
Dutta & Das (2002) and the developed model by Hazra & Das (2023) reveals a significant improvement in performance.



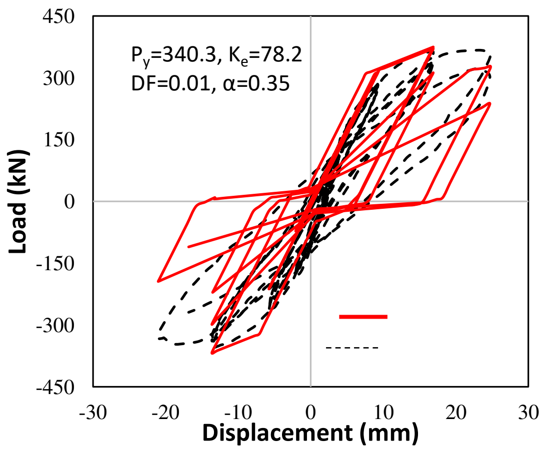
(a): [37] Sp-12: by Previous Model



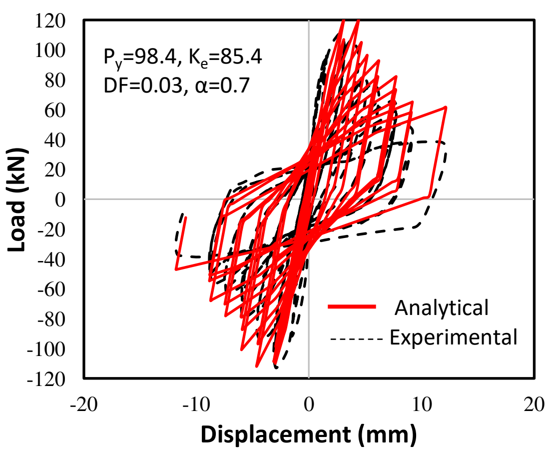
(b): *Tanaka and Park 1990:* Sp-6 by Developed Model



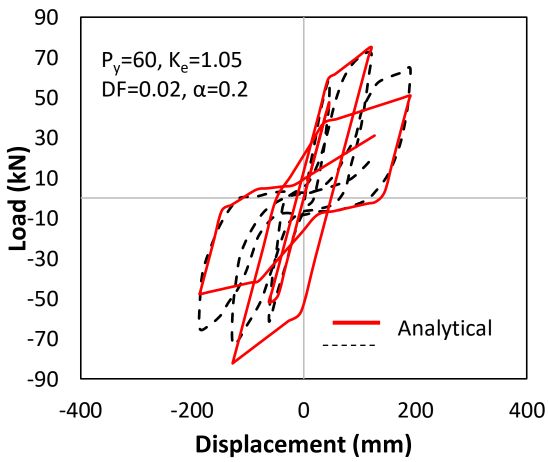
(c): *Gill et al.* Sp-2 by Previous Model



(d): *Lynn et al. 1996* Sp-3SMD12 by Developed Model



(e): *Nagasaka 1982* Sp- HPRC19-32 by Previous Model



(f): Unit-1 *Liu et al.* by Previous model

Fig. 14. Computationally reproduced hysteresis curve by proposed developed Uni-Axial Hysteresis Model

3. Conclusion

The following conclusions were drawn from the review of literatures reported herein on the hysteresis modelling of RC structural elements and also provided the insights of the proposed enhancement of simpler hysteresis model,

1. The complex hysteresis models which include mainly Bouc-Wen model and its derivatives can predict inelastic behavior relatively more accurate but due to the requirements of huge number of pre-calibrated lab-based test results as input parameters these types of models are very case specific.
2. On the other hand, simpler hysteresis model required few simple common parameters as input which are easier to calculate. These models make it relatively simple to conduct an all-encompassing investigation based on idealized structural system. On the basis of sample experimental data from cyclic load testing of related structural elements, the parameters might be derived.
3. The developed simpler hysteresis model to predict inelastic behavior of structural elements under cyclic loading has turned out to be a useful one. The hysteresis model by Hazra & Das (2023) has only four general input parameters, which are easily calculated in comparison with other existing sophisticated models. The previous drawbacks have been successfully eliminated by incorporating pinching effect and the effect of strain hardening. The introduction of new parameter stress ratio (α) has been justified through sensitivity analysis and established that for mere change in the value of 'α' there would be no significant changes in the response of structures. The performance analysis of the proposed hysteresis model shows less than 8% deviation from the experimental values at all cases.

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